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Topology optimization of reinforced concrete structures

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Publication date:
2011

Document Version
Publisher's PDF, also known as Version of record

Topology Optimization of Reinforced Concrete Structures

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EUROMECH 522, Erlangen, Germany, October 12 2011

Outline

1. Motivation
2. Modeling approach
3. Topology optimization
4. Preliminary results
5. Discussion

1 Motivation

2 Modeling approach

3 Topology optimization

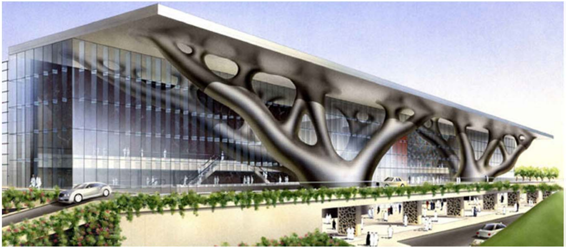
4 Preliminary results

5 Discussion

Topology optimization as an architectural design tool

The facade of the Qatar convention center

- Collaboration between architect (Arata Isozaki) and engineer (Mutsuro Sasaki).
- Topology generated using ESO.
- Actual structure built with tubes!



Optimization serves as inspiration for conceptual design

Topology optimization as an architectural design tool

Recent examples

Project UNIKABETON (2010):

Stromberg et al. (2011):

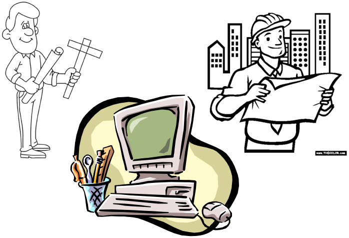
Topology optimization as an architectural design tool

- Architects are exploring ways of generating new forms.
- Advanced technology enables the production of complex structural forms in concrete.
- Architecture often determines the conceptual design, leaving little room for optimization.

The vision: enhance architect-engineer collaboration.

The challenges:

- Material characteristics of reinforced concrete: nonlinear, quasi-brittle in tension, different physical scales.
- Aesthetics $\neq$ Optimization.



Topology optimization as an architectural design tool

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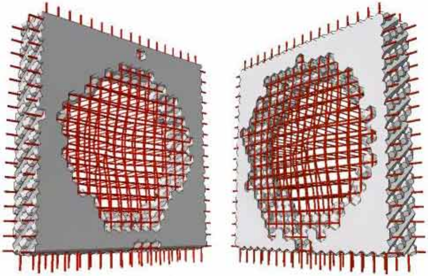


Design of structural concrete

Typical structures consist of 'B' and 'D' regions:

- B-regions: linear strain distribution; internal stress derived from sectional forces.
- D-regions: nonlinear strain distribution.

Leading design approach for cracked D-regions:

STRUT & TIE MODELING

(Marti 1985, Schlaich et al. 1987)

The forces are transferred via a truss-like structure consisting of struts in compression and ties in tension.

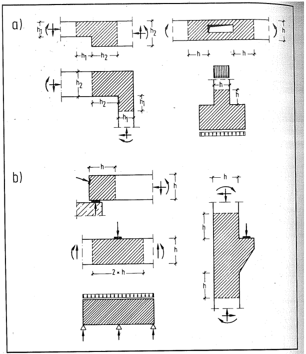
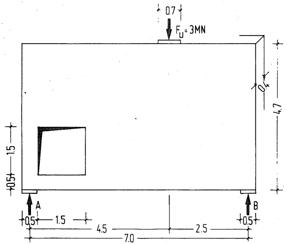
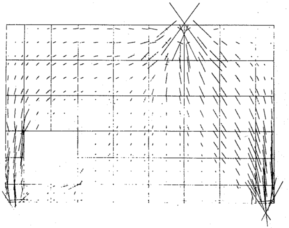


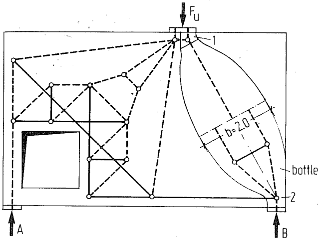
Fig. 1. D-regions (shaded areas) with nonlinear strain distribution due to (a) geometrical discontinuities; (b) statical and/or geometrical discontinuities.

Strut & tie approach (Schlaich et al. 1987)

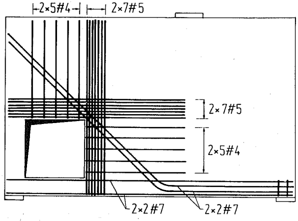


a) dimensions [m] and load





h) complete strut-and-tie-model



i) reinforcement

Generating S & T models using topology optimization

(Liang et al. 2000, Bruggi 2009, Moen and Guest 2010, Victoria et al. 2011)

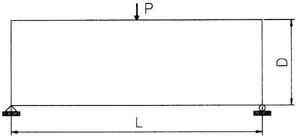
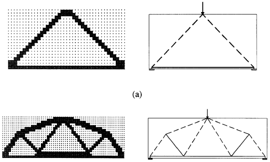
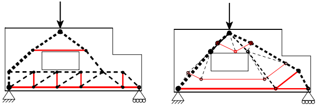
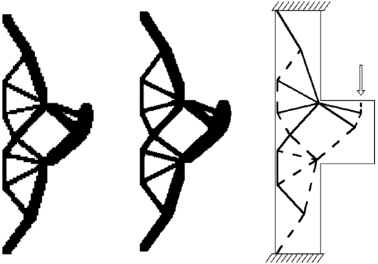


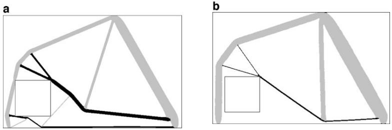
Fig. 10—Simply supported beams with various span-depth





(a) Traditional design (b) Minimum Compliance Design





Towards optimizing structural concrete

- Design is mostly based on linear elastic analysis.
- Topology optimization is mainly used for generating S & T models.
- The geometry of the concrete domain is considered fixed - S & T only provides reinforcement layout.
- Nonlinear FEA of reinforced concrete has progressed significantly over the last two decades.

The aim: design structural concrete based on topology optimization and nonlinear FEA.

- Conceptual design phase - enhance collaboration between architects and engineers.
- Optimize the use of concrete - reduce weight -> material consumption -> CO_2 emissions.
- Optimize the distribution of steel reinforcement in D-regions of the structure.

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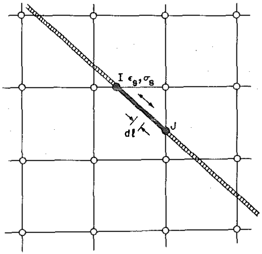
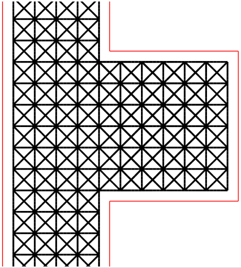
Embedded formulation

Reinforced concrete is modeled by the embedded formulation:

- Concrete is an isotropic continuum; material described by a gradient enhanced damage model.
- Steel reinforcement consists of 1-D bars; linear elastic behavior.
- Displacements of both phases are compatible using an embedded formulation (Phillips and Zienkiewicz 1976; Chang et al. 1987) ; bond-slip relation can also be considered (Balakrishnan and Murray 1986) .





Truss ground structure: all permissible rebars; embedded into the continuum concrete mesh.

Model for concrete

- Model based on "Gradient enhanced damage for quasi-brittle materials" (Peerlings et al. 1996) .
- Successfully applied recently for: multiphase material optimization of fiber reinforced composites (Kato et al. 2009); optimization of fiber geometry (Kato and Ramm 2010).

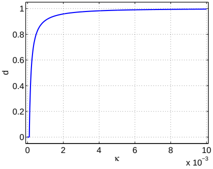
$$\begin{aligned}
\boldsymbol{\sigma} &= (1-d)\mathbf{C}\boldsymbol{\epsilon} \\
d &= d(\kappa) \\
\bar{\epsilon}_{eq} &\geq 0 \\
\dot{\kappa} \geq 0, \qquad \bar{\epsilon}_{eq} - \kappa &\leq 0, \qquad \dot{\kappa}(\bar{\epsilon}_{eq} - \kappa) = 0 \\
\bar{\epsilon}_{eq} - c\nabla^2\bar{\epsilon}_{eq} &= \epsilon_{eq}
\end{aligned}$$

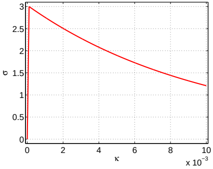
Model for concrete

Damage law

$$d = 1 - \frac{\kappa_0}{\kappa}\left(1 - \alpha + \alpha \exp^{-\beta(\kappa - \kappa_0)}\right)$$

(Mazars & Pijaudier-Cabot 1989)

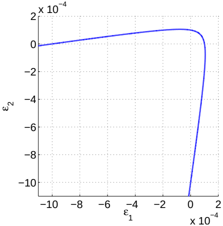




Equivalent strain

$$\epsilon_{eq} = \sqrt{3J_2} + mI_1$$

(Drucker-Prager function)



(example: $\epsilon_{eq} - \kappa_0 = 0$)

Finite element implementation

Typical Newton-Raphson iterative equation:

$$\begin{bmatrix} \mathbf{K}^{uu}_{i-1} + \mathbf{K}^{bars} & \mathbf{K}^{u\epsilon}_{i-1} \\ \mathbf{K}^{\epsilon u}_{i-1} & \mathbf{K}^{\epsilon\epsilon} \end{bmatrix} \begin{bmatrix} \delta \mathbf{u}_i \\ \delta \bar{\boldsymbol{\epsilon}}_{eq,i} \end{bmatrix} = \begin{bmatrix} \mathbf{f}^{u}_{ext} \\ \mathbf{f}^{\epsilon}_{i-1} \end{bmatrix} - \begin{bmatrix} \mathbf{f}^{u}_{int,i-1} + \mathbf{f}^{bar}_{int} \\ \mathbf{K}^{\epsilon\epsilon} \bar{\boldsymbol{\epsilon}}_{eq,i-1} \end{bmatrix}$$

With:

$$\mathbf{K}^{uu}_{i-1} = \int_\Omega \mathbf{B}^T (1 - d_{i-1}) \mathbf{C} \mathbf{B} \, d\Omega \qquad \mathbf{f}^{u}_{int,i-1} = \int_\Omega \mathbf{B}^T \boldsymbol{\sigma}_{i-1} d\Omega$$

$$\mathbf{K}^{u\epsilon}_{i-1} = -\int_\Omega \mathbf{B}^T \mathbf{C} \boldsymbol{\epsilon}_{i-1} q_{i-1} \tilde{\mathbf{N}} \, d\Omega \qquad \mathbf{f}^{\epsilon}_{i-1} = \int_\Omega \tilde{\mathbf{N}}^T \epsilon_{eq,i-1} d\Omega$$

$$\mathbf{K}^{\epsilon u}_{i-1} = -\int_\Omega \tilde{\mathbf{N}}^T \left(\frac{\partial \epsilon_{eq}}{\partial \boldsymbol{\epsilon}} \right)^T_{i-1} \mathbf{B} \, d\Omega \qquad q_{i-1} = \begin{cases} \left(\frac{\partial d}{\partial \kappa} \right)_{i-1} & \bar{\epsilon}_{eq,i-1} > \kappa_{old} \\ 0 & \bar{\epsilon}_{eq,i-1} \le \kappa_{old} \end{cases}$$

$$\mathbf{K}^{\epsilon\epsilon} = \int_\Omega \left(\tilde{\mathbf{N}}^T \tilde{\mathbf{N}} + \tilde{\mathbf{B}}^T c \tilde{\mathbf{B}} \right) d\Omega$$

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Problem formulation - rebar optimization only

$$\begin{aligned}
\min_{\mathbf{x}} \phi(\mathbf{x}) &= -\theta_{Nincr} \hat{f}^p u^p_{Nincr} \\
\text{s.t.:} \quad & \sum_{i=1}^{N_{bars}} a_i l_i \leq \rho V_{domain} \\
& 0 \leq x_i \leq 1, \qquad i = 1, ..., N_{bars} \\
\text{with:} \quad & \mathbf{R}_n(\mathbf{u}_n, \theta_n, \bar{\boldsymbol{\epsilon}}_{eq,n}, \boldsymbol{\kappa}_{n-1}, \mathbf{x}) = 0 \qquad n = 1, ..., Nincr \\
& \mathbf{H}_n(\bar{\boldsymbol{\epsilon}}_{eq,n}, \boldsymbol{\kappa}_n, \boldsymbol{\kappa}_{n-1}) = 0 \qquad n = 1, ..., Nincr - 1
\end{aligned}$$

Design parameterization:

- $a_i = a_{min} + (a_{max} - a_{min})x_i$
- a_{max} related to geometry and desired rebars.
- $\mathbf{K}^{bars} = \sum_{i=1}^{N_{bars}} E_{bar}(a_{min} + (a_{max} - a_{min})x_i^{p_{bar}})\mathbf{K}_i^0$

Problem formulation - concrete & rebar optimization

$$\begin{aligned}
\min_{\mathbf{x}} \phi(\mathbf{x}) \quad &= \quad \frac{\sum_{i=1}^{N_{elem}} \tilde{x}_i}{N_{elem}} \\
\text{s.t.:} \quad & -\theta_{Nincr} \hat{f}^p u^p_{Nincr} + g^\star \leq 0 \\
& \sum_{i=1}^{N_{bars}} a_i l_i \leq \rho V_{domain} \\
& 0 \leq x_i \leq 1, \qquad i = 1, ..., (N_{elem} + N_{bars}) \\
\text{with:} \quad & \mathbf{R}_n(\mathbf{u}_n, \theta_n, \bar{\boldsymbol{\epsilon}}_{eq,n}, \boldsymbol{\kappa}_{n-1}, \mathbf{x}) = 0 \qquad n = 1, ..., Nincr \\
& \mathbf{H}_n(\bar{\boldsymbol{\epsilon}}_{eq,n}, \boldsymbol{\kappa}_n, \boldsymbol{\kappa}_{n-1}) = 0 \qquad n = 1, ..., Nincr - 1
\end{aligned}$$

Design parameterization, SIMP approach for concrete:

- $E_i = E_{min} + (E_{max} - E_{min})\tilde{x}_i^{p_E}$
- $E_{max} = 30000$ [MPa], $E_{min} = 3000$ [MPa]
- All other material properties are considered constant.

Adjoint sensitivity analysis

The augmented objective functional

$$\begin{aligned}\hat{\phi}(\mathbf{x}) &= -\theta_{Nincr}\hat{f}^p u^p_{Nincr} - \sum_{n=1}^{Nincr} \boldsymbol{\lambda}_n^T \mathbf{R}_n(\mathbf{u}_n, \theta_n, \bar{\boldsymbol{\epsilon}}_{eq,n}, \boldsymbol{\kappa}_{n-1}, \mathbf{x}) \\ &\quad - \sum_{n=1}^{Nincr-1} \boldsymbol{\gamma}_n^T \mathbf{H}_n(\bar{\boldsymbol{\epsilon}}_{eq,n}, \boldsymbol{\kappa}_n, \boldsymbol{\kappa}_{n-1})\end{aligned}$$

The explicit design sensitivities

$$\frac{\partial \hat{\phi}_{exp}}{\partial x} = -\sum_{n=1}^{Nincr} \boldsymbol{\lambda}_n^T \frac{\partial \mathbf{R}_n}{\partial x}$$

Due to path dependency, a backwards-incremental procedure is required for computing the adjoint variables (following the framework by Michaleris et al. 1994).

Adjoint sensitivity analysis

Solve for adjoint $\boldsymbol{\lambda}_{Nincr}$ in the final increment

$$\begin{aligned} \mathbf{K}_{Nincr}^T \boldsymbol{\lambda}_{Nincr} &= \mathbf{0} \qquad \text{(non-prescribed DOF)} \\ \hat{\mathbf{f}}^T \boldsymbol{\lambda}_{Nincr} &= -\hat{f}^p u_{Nincr}^p \end{aligned}$$

Compute adjoint $\boldsymbol{\gamma}_{Nincr-1}$ related to path-dependency

$$\boldsymbol{\gamma}_{Nincr-1} = -\left(\frac{\partial \mathbf{R}_{Nincr}}{\partial \boldsymbol{\kappa}_{Nincr-1}} \right)^T \boldsymbol{\lambda}_{Nincr}$$

Solve for adjoint $\boldsymbol{\lambda}_{Nincr-1}$

$$\mathbf{K}_{Nincr-1}^T \boldsymbol{\lambda}_{Nincr-1} = -\tilde{\mathbf{N}} \frac{\partial \mathbf{H}_{Nincr-1}}{\partial \bar{\boldsymbol{\epsilon}}_{eq,Nincr-1}} \boldsymbol{\gamma}_{Nincr-1} \qquad \text{(non-prescribed DOF)}$$

Compute adjoint $\boldsymbol{\gamma}_{Nincr-2}$

$$\boldsymbol{\gamma}_{Nincr-2} = -\left(\frac{\partial \mathbf{R}_{Nincr-1}}{\partial \boldsymbol{\kappa}_{Nincr-2}} \right)^T \boldsymbol{\lambda}_{Nincr-1} - \frac{\partial \mathbf{H}_{Nincr-1}}{\partial \boldsymbol{\kappa}_{Nincr-2}} \boldsymbol{\gamma}_{Nincr-1}$$

Continue until $\boldsymbol{\gamma}_1$, $\boldsymbol{\lambda}_1$.

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Implementation aspects

- NL-FEA by a displacement-controlled Newton-Raphson procedure with automatic incrementation.
- Regularization of the concrete phase by a volume-preserving projection function (Xu et al. 2010) on top of a density filter (Bruns and Tortorelli 2001; Bourdin 2001) .
- Gradual penalization, typically up to 3.0 for the concrete phase and 1.1-1.5 for the rebars.
- Design update by MMA (Svanberg 1987) .

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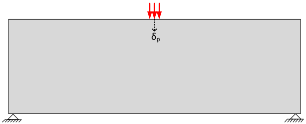
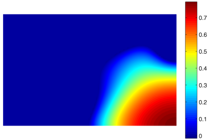
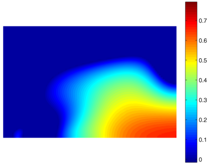
- **ONLY PRELIMINARY RESULTS**

Deep beam

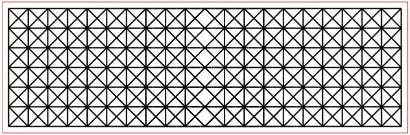
Result with $\rho = 0.005$





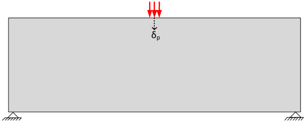


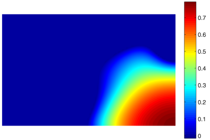
Iter: 1 Objval: −2.958e−005 Constval: −8.742e−007

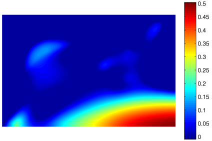


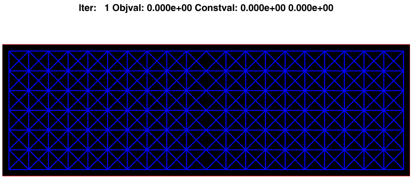
Deep beam

Result with $\rho = 0.005, g^{\star} \approx 0.8 \times \phi_{rebaronly}$



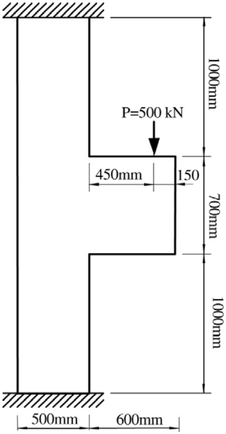


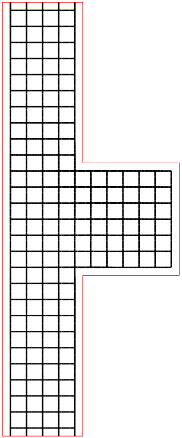


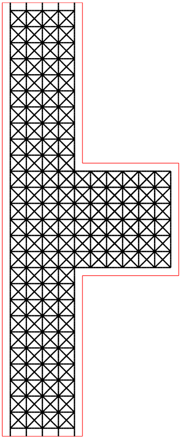


Corbel

Kwak and Noh 2006; Bruggi 2009; Victoria et al. 2011







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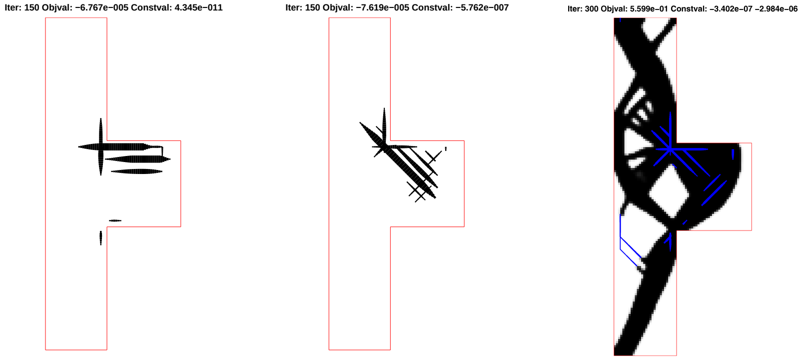
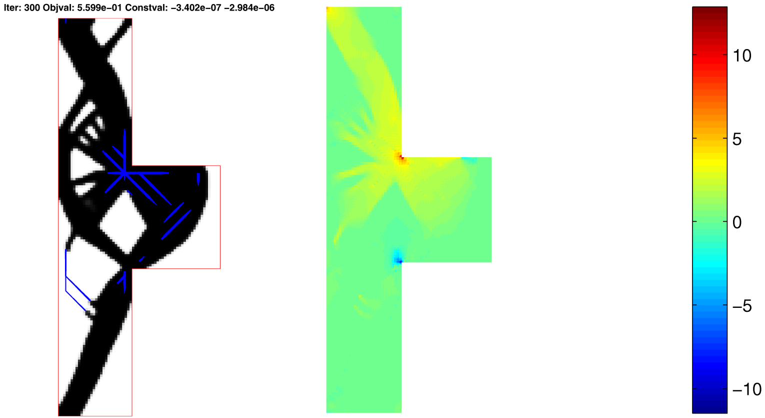
Results with $\rho = 0.005, g^{\star} \approx 0.8 \times \phi_{rebaronly}$



Corbel

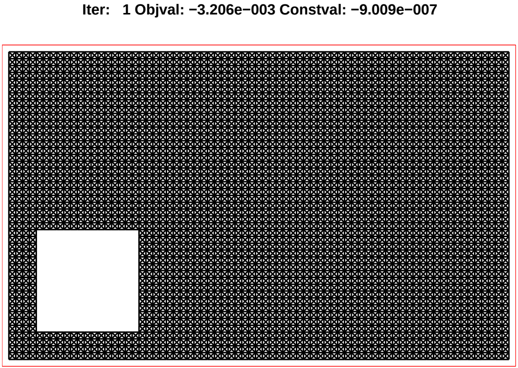
Optimized design utilizes concrete in tension



Optimized design and σ_1

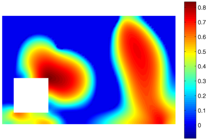
Wall with opening

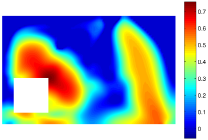
Result with $\rho = 0.005$

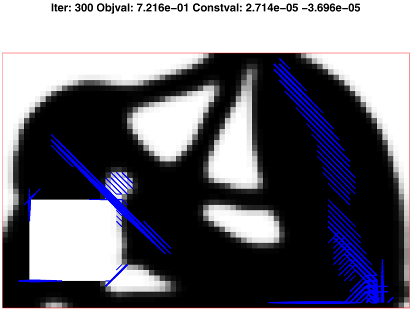


Wall with opening

Result with $\rho = 0.005, g^{\star} \approx 0.9 \times \phi_{rebaronly}$







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Remarks regarding preliminary results

- Single computational procedure for determining the reinforcement layout.

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Remarks regarding preliminary results

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- Single computational procedure for determining the reinforcement layout.
- ‘Cutting out’ unnecessary concrete is possible, s.t. a compliance constraint; helps identify an optimized S & T model.
- Practical requirements can be taken into account, e.g. clear cover, rebar spacing, minimum reinforcement, allowed deflections etc.
- Extreme concrete principal stresses can be checked immediately.
- Integration into standard FEA packages seems straightforward.

Discussion

Thoughts for the near future

- Refine results and study the influence of:
  1. De-localization parameter c.
  2. Damage law parameters.
  3. Equivalent strain measure.
  4. Reinforcement ground structure.

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- Compare to linear-elastic optimization and conventional S&T.
- Replace 'soft' concrete with void; avoid uncovered bars.

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  4. Reinforcement ground structure.
- Compare to linear-elastic optimization and conventional S&T.
- Replace ‘soft’ concrete with void; avoid uncovered bars.
- Improve concrete modeling: plastic-damage (Lubliner et al. 1989) ; plastic-fracture (Cervenka et al. 1998) .

Discussion

Thoughts for the near future

- Refine results and study the influence of:
  1. De-localization parameter c.
  2. Damage law parameters.
  3. Equivalent strain measure.
  4. Reinforcement ground structure.
- Compare to linear-elastic optimization and conventional S&T.
- Replace 'soft' concrete with void; avoid uncovered bars.
- Improve concrete modeling: plastic-damage (Lubliner et al. 1989) ; plastic-fracture (Cervenka et al. 1998) .
- Consider objective functions related to extent of fracture / damage.

Acknowledgments

Thanks for discussing: Ole Sigmund, Claus Pedersen.

Thanks for the Fortran-MMA code: Krister Svanberg.

Project funded by the Danish Council for Independent Research | Technology and Production Sciences.